Algebra 2

1-01 Solve Linear Systems of Equations and Inequalities by Graphing

System of equations

- More than one _____ that share the _____ solution.
- Often, they involve more than one _____.
- In order to solve them, you need _____ equations as there are _____.

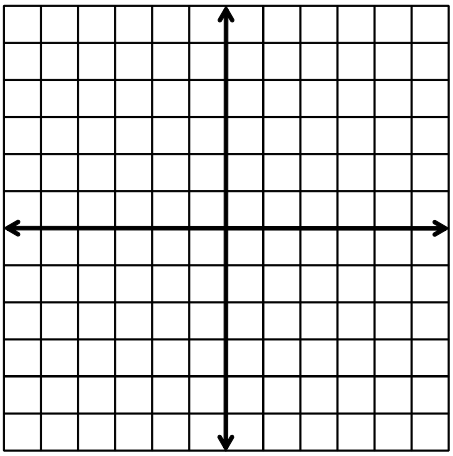
Solutions to systems

- An _____ that works in _____ equations.
- Solutions are where the graphs _____.

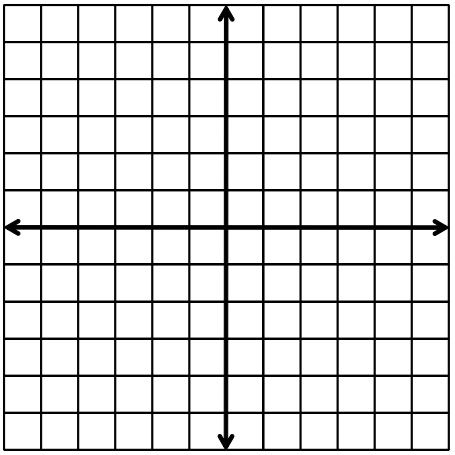
Solve by graphing

1. Graph both equations on the _____ graph.
2. Where they cross is the _____.

Solve by graphing $\begin{cases} 3x + 2y = -4 \\ x + 3y = 1 \end{cases}$



Solve by graphing $\begin{cases} 3x - 2y = 10 \\ 3x - 2y = 2 \end{cases}$



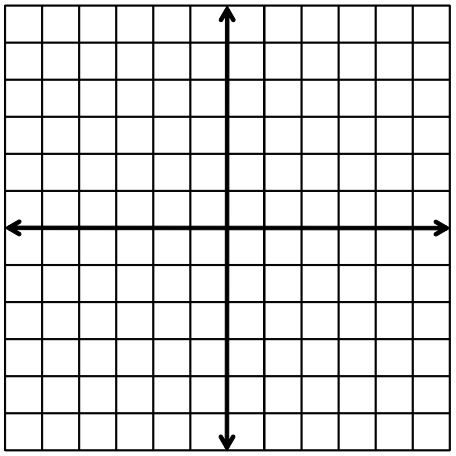
Algebra 2 1-01 Name: _____

To solve systems of inequalities

- Graph them all on _____graph.
- Solution is where all graphs _____.

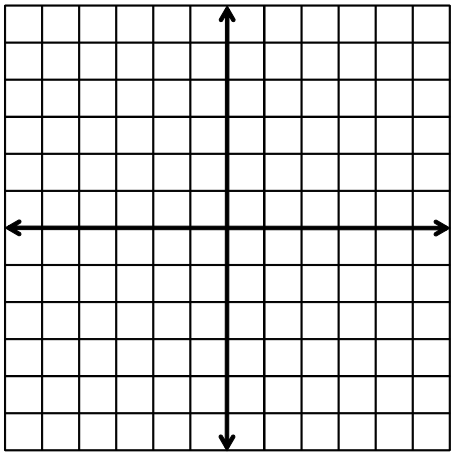
Solve the system of inequalities

$$\begin{cases} x \geq 2 \\ x + y < 3 \end{cases}$$



Solve the system of inequalities

$$\begin{cases} y < -\frac{4x}{5} - 4 \\ y > -\frac{4x}{5} + 2 \end{cases}$$



Algebra 2

1-02 Solve Linear Systems Algebraically

Substitution

1. Solve one equation for _____ variable
2. Use that expression to _____ that variable in the _____ equation
3. _____ the new equation
4. _____ back into the _____ equation
5. _____ for the second variable

Solve $\begin{cases} y = x + 2 \\ 2x + y = 8 \end{cases}$

Solve $\begin{cases} 3x + 2y = 8 \\ x + 4y = -4 \end{cases}$

Elimination

1. _____ up the equations into _____
2. Multiply _____ or _____ equations by numbers so that one variable has the same _____ , but opposite _____
3. _____ the equations
4. _____ the resulting equation
5. _____ the value into one _____ equation and solve

Solve $\begin{cases} 2x - 3y = -14 \\ 3x - y = -7 \end{cases}$

Solve $\begin{cases} 3x + 11y = 4 \\ -2x - 6y = 0 \end{cases}$

Number of Solutions

- If _____ variables _____ after you substitute or combine and
- You get a _____ statement like 2 = 2 → _____ solutions
- You get a _____ statement like 2 = 5 → _____ solution

Summary of Solving Techniques

- When to graph?
  - To get _____ picture and _____
- When to use substitution?
  - When _____ of the coefficients is 1
- When to use elimination?
  - When _____ of the coefficients is 1

Worksheet

Algebra 2

1-03 Solve Linear Systems in Three Variables

- Linear equation in 3 variables graphs a _____

Solution to system in 3 variables

- Ordered _____ (_____)

Is $(2, -4, 1)$ a solution of $\begin{cases} x + 3y - z = -11 \\ 2x + y + z = 1 \\ 5x - 2y + 3z = 21 \end{cases}$

Elimination Method

Like two variables, you just do it _____ once.

1. Combine _____ and _____ to eliminate a variable
2. Combine _____ and _____ to eliminate the _____ variable as before
3. Combine these _____ equations to find the _____ variables
4. Substitute those _____ variables into one of the _____ equations to get the _____ variable

- If you get a _____ statement like 8 = 0 → _____ solution
- If you get an _____ like 0 = 0 → _____ solutions

Solve $\begin{cases} 2x + 3y + 7z = -3 \\ x - 6y + z = -4 \\ -x - 3y + 8z = 1 \end{cases}$

Solve $\begin{cases} -x + 2y + z = 3 \\ 2x + 2y + z = 5 \\ 4x + 4y + 2z = 6 \end{cases}$

Solve $\begin{cases} x + y + z = 6 \\ x - y + z = 6 \\ 4x + y + 4z = 24 \end{cases}$

If there are infinitely many solutions

- Let _____ (Use *x*, *y*, or *z* based on what is convenient)
- Solve for _____ in terms of _____
- Substitute those to find _____ in terms of _____
- Sample answer _____

You have $1.42 in quarters, nickels, and pennies. You have twice as many nickels as quarters. You have 14 coins total. How many of each coin do you have?

32 #1, 5, 9, 15, 17, 19, 23, 43, 47, 51, 53, 55 = 12 (You can solve them all by elimination if you want.)

Algebra 2

1-04 Perform Basic Matrix Operations (12.1)

Matrix

- _____ arrangement of things (variables or numbers in math)

$$\begin{bmatrix} 2 & -1 & 5 & a \\ 2 & y & 6 & b \\ 3 & 14 & x & c \end{bmatrix}$$

- Dimensions
  - _____ by _____
  - _____ for the above matrix
- In order for two matrices to be equal, they must be the _____ dimensions and _____ elements must be the _____

$$\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

Find the variables $\begin{bmatrix} 2 & y+1 \\ x/3 & 4 \end{bmatrix} = \begin{bmatrix} w & -4 \\ 5 & z-4 \end{bmatrix}$

Adding and Subtracting

- You can only add and subtract matrices that are the _____
- When you add or subtract, add the _____ elements.

$\begin{bmatrix} 1 & 2 \\ -5 & 4 \end{bmatrix} + \begin{bmatrix} -2 & 5 \\ 4 & -3 \end{bmatrix}$

$[2 \quad -3] - [3 \quad 4] + [1 \quad 0]$

$\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} 0 & 3 & 1 \\ 2 & 5 & 2 \end{bmatrix}$

Scalar Multiplication

- Multiply each element by the _____
- _____

$$3\begin{bmatrix} 5 & -2 & 7 \\ -3 & 8 & 4 \end{bmatrix}$$

The National Weather Service keeps track of weather.

June 2014	Benton Harbor	South Bend
Precip Days	13	18
Clear Days	16	13
Ab Norm T	12	19

July 2014	Benton Harbor	South Bend
Precip Days	14	15
Clear Days	18	18
Ab Norm T	2	8

What is meaning of the first matrix + second matrix?

Use matrix operations to find the total weather stats of each city.

650 #1, 5, 9, 13, 15, 17, 19, 21, 23, 25, 29, 33, 35, 37, 39, and Mixed Review = 20

Algebra 2

1-05 Multiply Matrices (12.2)

Matrix Multiplication

- Matrix multiplication can only happen if the number of _____ of the _____ matrix is the same as the number of _____ on the _____ matrix.
- You can multiply a 3×5 with a 5×2.
- 3×**5** · **5**×2 → _____ will be the dimensions of the answer
- Because of this _____!

$\begin{bmatrix} 1 & 2 \\ 0 & -3 \end{bmatrix} \cdot \begin{bmatrix} -2 & 1 \\ 4 & 3 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 4 \\ -2 & 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 3 \\ 5 \end{bmatrix}$

Use the given matrices to evaluate $2(AC) + B$

$A = \begin{bmatrix} 5 & -9 \\ -1 & 3 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 4 \end{bmatrix}, C = \begin{bmatrix} 2 \\ -6 \end{bmatrix}$

The members of two bowling leagues submit meal choices for an upcoming banquet as shown. Each pizza meal costs $16, each spaghetti meal costs $22, and each Sam's chicken meal costs $18. Use matrix multiplication to find the total cost of the meals for each league.

	Pizza	Spaghetti	Sam's Chicken
League A	18	35	7
League B	6	40	9

658 #1, 5, 7, 9, 11, 13, 15, 17, 21, 23, 25, 29, 35, 39, 41, and Mixed Review = 20

Algebra 2

1-06 Evaluate Determinants (12.3)

Determinant

- Number associated with _____ matrices
- Symbolized by _____ or _____

Determinant of 2×2 matrix

- Multiply along the _____ diagonal and _____ the product of the _____ diagonal.

$$\begin{vmatrix} 2 & -1 \\ 3 & 4 \end{vmatrix}$$

Determinant of 3×3 Matrix

- Copy the first _____ behind the matrix and then _____ the products of the _____ diagonals and _____ the product of the _____ diagonals.

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$$

Area of a Triangle

$$Area = \pm\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

where *x*'s and *y*'s are the coordinates of the _____

Find the area of a triangle with vertices of (2, 4), (5, 1), and (2, −2)

Cramer's Rule

1. Write the equations in _____ form
2. Make a matrix out of the _____

2×2 System

$ax + by = e$
$cx + dy = f$ gives $x = \frac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}, y = \frac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$

$2x + y = 1$
$3x - 2y = -23$

3×3 System

- Same as _____ system
- The denominator is the determinant of the _____ matrix and the numerator is the _____ only with the column of the _____ you are solving for replaced with the _____.

$2x - y + 6z = -4$
$6x + 4y - 5z = -7$
$-4x - 2y + 5z = 9$

667 #1, 3, 5, 9, 13, 15, 17, 21, 23, 29, 31, 33, 35, 39, 45, and Mixed Review = 20

Algebra 2

1-07 Use Inverse Matrices to Solve Linear Systems (12.4)

Identity Matrix

The Identity Matrix _____ with any matrix of the _____ dimension equals the _____ matrix.

$A \cdot I = I \cdot A =$ _____

This is the matrix equivalent of 1

$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ $\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$

You cannot _____ by a matrix!

So we _____ by the _____ of a matrix.

$A \cdot A^{-1} =$ _____

If A, B, and X are matrices, and

$A \cdot X = B$

$A^{-1} \cdot A \cdot X = A^{-1} \cdot B$

$I \cdot X = A^{-1} \cdot B$

$X =$ _____

Inverse Matrix

The Rule for 2×2

If $A = \begin{bmatrix}a & b\\c & d\end{bmatrix}$, then $A^{-1} = \frac{1}{\begin{vmatrix}a & b\\c & d\end{vmatrix}}\begin{bmatrix}d & -b\\-c & a\end{bmatrix}$

$\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}^{-1}$

$\begin{bmatrix}-2 & -1\\4 & 0\end{bmatrix}^{-1}$

Solve a matrix equation

$AX = B$

$$\begin{bmatrix} -3 & 4 \\ 5 & -7 \end{bmatrix} X = \begin{bmatrix} 3 & 8 \\ 2 & -2 \end{bmatrix}$$

Solve a system of linear equations

$$\begin{aligned} 2x + \ y &= -13 \\ x - 3y &= \ \ 11 \end{aligned}$$

676 #1, 5, 9, 11, 13, 15, 17, 27, 29, 31, and Mixed Review = 15

Algebra 2

1-Review

Take this test as you would take a test in class. When you are finished, check your work against the answers.

1-01

Graph the system and estimate the solution.

1. $\begin{cases} y = \frac{2}{3}x + 1 \\ y = -\frac{1}{2}x - \frac{5}{2} \end{cases}$

2. $\begin{cases} 2x + y = 3 \\ x - y = 0 \end{cases}$

Graph the system of inequalities.

3. $\begin{cases} y < 2x + 1 \\ y \geq -x - 2 \end{cases}$

1-02

Solve the system algebraically.

4. $\begin{cases} y = x + 2 \\ 2x - 2y = 3 \end{cases}$

5. $\begin{cases} 3x - 2y = -7 \\ x + 2y = 11 \end{cases}$

6. Jim has two jobs. The first week he works 2 hours at job A and 3 hours at job B and earns \$57.50. The second week he works 5 hours at job A and 2 hours at job B and earns \$75. What is his pay rate at job A?

7. How do you know if there are many solutions when you are solving algebraically?

1-03

Is the given point a solution to the system?

8. $\begin{cases} x - y + 2z = -7 \\ y - 3z = 11 \\ x + z = -2 \end{cases}$; point $(1, 2, -3)$

Solve the system algebraically.

9. $\begin{cases} x + y + z = 4 \\ -x + y - 2z = -4 \\ -2y - z = -4 \end{cases}$

10. What does the graph of a linear equation in three variables look like?

1-04

Simplify.

11. $\begin{bmatrix} 1 & 8 \\ -3 & 5 \end{bmatrix} - \begin{bmatrix} -2 & 0 \\ -9 & -4 \end{bmatrix}$

12. $3\begin{bmatrix} 2 & 8 \end{bmatrix}$

13. $2\begin{bmatrix} 3 \\ -4 \end{bmatrix} + \begin{bmatrix} 1 \\ 5 \end{bmatrix}$

1-05

Simplify.

14. $\begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} -2 & 3 \\ -1 & 4 \end{bmatrix}$

15. $\begin{bmatrix} 1 & 2 \\ -2 & -1 \end{bmatrix}\begin{bmatrix} 3 & -3 \\ 1 & -1 \end{bmatrix}$

16. How do you know if two matrices can be multiplied?

1-06

Evaluate the determinant.

17. $\begin{vmatrix} 3 & -1 \\ 2 & 7 \end{vmatrix}$

18. $\begin{vmatrix} 1 & 3 & 0 \\ -2 & -1 & 2 \\ 4 & 0 & -1 \end{vmatrix}$

19. Find the area of the triangle with vertices $(1, 2)$, $(0, -2)$, $(3, 1)$.

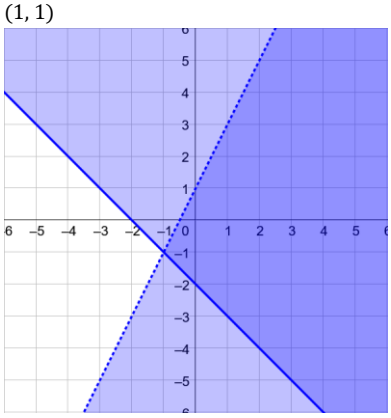
1-07

20. What is the product of a matrix with its inverse?

21. Find inverse of $\begin{bmatrix} 2 & 1 \\ 1 & -3 \end{bmatrix}$.

22. Use an inverse to solve $\begin{cases} 2x + y = 8 \\ x - 3y = -3 \end{cases}$.

Answers

1. $(-3, -1)$
2. $(1, 1)$
3. 
4. No solution
5. $(1, 5)$
6. \$10 per hour
7. All variables are eliminated and the result is a true statement.
8. Yes
9. $(1, 1, 2)$
10. A plane
11. $\begin{bmatrix} 3 & 8 \\ 6 & 9 \end{bmatrix}$
12. $[6 \quad 24]$
13. $\begin{bmatrix} 7 \\ -3 \end{bmatrix}$
14. $[-4 \; 11]$
15. $\begin{bmatrix} 5 & -5 \\ -7 & 7 \end{bmatrix}$
16. The number of columns in the 1st matrix = number of rows in the 2nd matrix
17. 23
18. 19
19. $\frac{9}{2}$
20. Identity matrix
21. $\begin{bmatrix} \frac{3}{7} & \frac{1}{7} \\ \frac{1}{7} & -\frac{2}{7} \end{bmatrix}$
22. $(3, 2)$